

EFFECT OF SONIC BOOM ON STRUCTURES
THIRD REPORT
MEASUREMENT OF EIGENFREQUENCIES OF BUILDING
STRUCTURES WHICH ARE SENSITIVE TO THE "BOOM"

P. De Tricaud

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EFFECT OF SONIC BOOM ON STRUCTURES
THIRD REPORT
MEASUREMENT OF EIGENFREQUENCIES OF BUILDING
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P. De Tricaud

ABSTRACT. The first eigenfrequency, damping and dispersions were measured for various types of full-sized building partitions and ceilings. Panels were made of plaster, plaster tile, plaster brick and various sandwich constructions. Measurements were compared with theoretical predictions. Various panel fixation methods were evaluated.

I. Goal of the Study

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This study falls within the framework of research on the effects of "sonic booms" on buildings. It is applicable to light structures (partitions, ceilings, glass panes, roofing elements) which are subject to wear.

It is well known that the response of a plate to an overpressure in the shape of an impulse depends on its eigenfrequencies. In particular, in the case of excitations by a "sonic boom", American studies have shown that the forces produced, caused exclusively by the response of the first vibration mode, are overwhelming. This is especially true if the period of the first mode is essentially equal to the total duration of the "sonic boom". This consists of a phenomenon which can be compared with resonance in the presence of large amplitudes.

* Numbers in the margin indicate pagination in the original foreign text.

⁽¹⁾ Convention No. 69-34-412-00-480-75-01, February 1, 1971

The sonic boom can have a direct effect (case of a sonic boom impinging on a closed window), or an indirect effect by penetrating the interior of the building through an opening. In the first case, pressure variations occur in the shape of damped sinusoids which excite the eigenmodes of light components in the interior of the room (example: light partitions and ceilings).

It is therefore imperative to know the dynamic characteristics of the various light components of the building if it is desired to evaluate the risks incurred by a building subjected to sonic booms, or if it is desired to build an "immune" construction.

The goal of this measurement series is the following:

- to first obtain an idea of the order of magnitude of the first eigenfrequency. If possible, we will attempt to determine the damping of the given light elements, taking their dimensions into account.

- to know the dispersion of the first eigenfrequency for a type of light element. Knowledge of this dispersion is essential if one wants to estimate the percentage of buildings of a given type which could be damaged after sonic booms. This dispersion is brought about by:

- the dispersion of the modulus of elasticity and the density of the materials forming the light elements. In particular, the average modulus of elasticity of a partition can vary over a wide range if the elements which make it up are not manufactured in a consistent manner. The module can also vary depending on the "age" of the material (especially the plaster).

- the method of fixation of the light elements to the rigid parts of the building. It is known that the first eigenfrequency of a rectangular plate can vary considerably depending on the nature of the boundary conditions: free edges, clamped edges, rigidly or elastically

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supported, or intermediate cases among these. The four edges can also not be fixed in the same way. When the installation mode is the same for a given type of partition, there can nevertheless be differences in the boundary conditions which are the result of the good or bad placing of the light element.

- to compare the experimental results with theoretical calculations in order to have an approximation of the first eigenfrequency of a light element which is sensitive to the sonic boom without carrying out measurements.

This latter objective is more difficult. It will be seen that numerous difficulties must be overcome

II. Course of the Study

II-1. Properties of Buildings which can be Sensitive to Sonic Boom

A distinction should be made between external walls of the building and interior walls.

Among the exterior walls, the following elements are sensitive to the boom:

- the glass panes: glass panes of windows or single large panels (display windows of stores, airport reception areas...)

- roofs: It should be noted that roofs are frequently damaged by explosions of a lesser or higher intensity. In particular, the tiles can become detached from their support and fall on the ground.

/3

There are difficulties connected with the study of the dynamic characteristics of a roof. The response of a roof to a sonic boom also depends on the hermeticity of the roof with respect to the overpressure. We are awaiting experimental results conducted at Istres before we will undertake a group of systematic measurements of roofs.

Among the interior walls, the following constitute the sensitive elements:

- light partitions, which separate two interior rooms of a building.

We carried out measurements on three types of partitions:

- prefabricated "Pleinclam" plaster tile partitions

- partitions made of "Placopan" (two Placoplâtre panels connected by a honeycomb)

- partitions made up of an assembly of plaster bricks (crossed bricks) which are connected by plaster, with a coat of plaster on both sides of the partition

- light ceilings: These elements are particularly sensitive to the sonic boom and have already been mentioned in complaints. We will attempt to measure the first eigenfrequency of several identical suspended ceilings. This will be the object of a series of future studies.

II-2. Method of Measurement

The movement of the partition and the glass pane is studied only at the center; the method used is the following:

The partition is set into motion by the percussion (at its center) of a tennis ball which is thrown by hand and caught before it touches the ground, so that any perturbation of the signal is avoided.

A 43.30 Bruel and Kjaer accelerometer is attached on the two sides by means of adhesive film at the center of the side opposite the one which is hit by the tennis ball. Preliminary tests have shown that this method of fixation is usable with an error of ± 0.5 dB between 20 and 3,000 Hz.

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The output of the accelerometer is connected with the Bruel and Kjaer 26.16 preamplifier (gain of 1) followed by a direct current amplifier having a gain of ten. Its output is connected with a Tolana 466 frequency modulation magnetic recorder having a pass band between 0 to 300 Hz.

For analysis in the laboratory, the recorded signals on the magnetic band are transcribed by means of an appropriate amplifier. A $1/3$ octave band filter is interposed. It is transcribed onto a ACBA0300 pen recorder having a linear frequency response between 0 and 100 Hz.

The first analysis is made without interposition of the filter, which results in a coarse value of the first eigenfrequency. The central frequency of the filter is then adjusted for this value: one then has a damped sinusoid in which all harmonics are eliminated.

For each partition, two recordings are made (two throws of the tennis ball). The eigenfrequency is calculated from a time corresponding to ten periods in order to increase the accuracy. Using these two recordings from the same partition, eigenfrequencies which were very close were always obtained (deviation of the frequency rarely exceed 1%).

The damping is measured from the decrease in the vibration amplitude of the partition.

The dimensionless damping α of the first mode is given by the formula

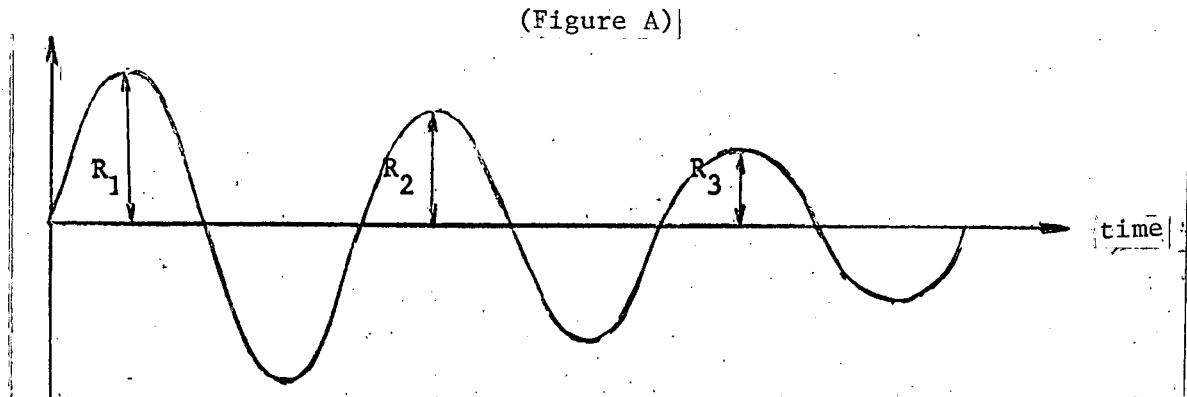
$$\alpha = 0,366 \log \frac{R_1}{R_2}$$

where, if α is calculated for various periods, which is more accurate:

$$\alpha = 0,366 \frac{1}{n} \log \frac{R_1}{R_{(1+n)}}$$

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Acceleration at the center of the panel



The curve utilized is the one given by the U.V. paper recorder with the interposed filter.

III-3. Experimental Results

a - "Pleinclam" Plaster Tile Partition

These plaster tiles are flat and have the following dimensions:

- thickness 7 cm
- length 66 cm
- height ~50 cm

They are fabricated in industry and are assembled by glueing to make up a complete partition.

Special glues ("Clam" and "Sceljoint" glues) are used to glue the partitions into large structures.

The measurements were made in a 15 story apartment house, with three rooms per apartment. Because of work in progress, only 26 partitions could be studied located between the third and ninth floor, inclusive.

The dimensions of the partition studied are the following:

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- height 2.50 m
- length 4.40 m

Figure 1 shows the dispersion of the various eigenfrequencies. It can be seen that the eigenfrequencies seem to diminish as one progresses up to the higher floors. No explanation could be given for this phenomenon.

The calculation shows that:

- the arithmetic mean of the eigenfrequencies is 21.9 Hz
- the mean square deviation σ (or standard deviation) is equal to 4.5 Hz.

Figure 2 shows the statistical distribution of the various eigenfrequencies. It can be seen that this distribution is essentially Gaussian (line on the graph).

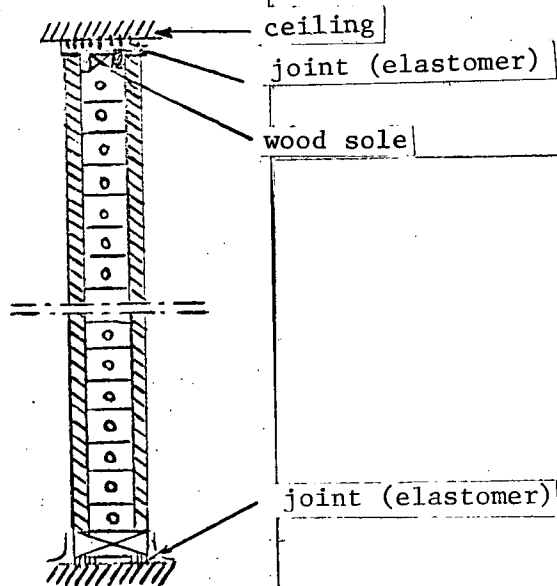
b - "Placopan" Partition

This prefabricated partition consists of an assembly of panels having a height of 3.5 m and a length of 1.20 m which are glued together and covered with craft paper at the joint. It is constructed out of two Placoplâtre walls a distance of 4 cm apart and connected by crossed cardboard plates with a thickness from 1 to 2 mm (see diagram, page 15^{*}). ✓

This partition is connected with the building as follows:

* Translator's Note: English page 23.

1
2
3
4
5



The measurements were carried out in a residential building having eight floors with two apartments per floor.

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Each apartment contained:

- three partitions connected with the structure along the edges and having the dimensions:

- length 3.15 meters
- height 2.50 meters

We will call these partitions type 1

- a partition connected with the structure along three edges, the fourth edge is used to affix the door (see diagram) and has the dimensions:

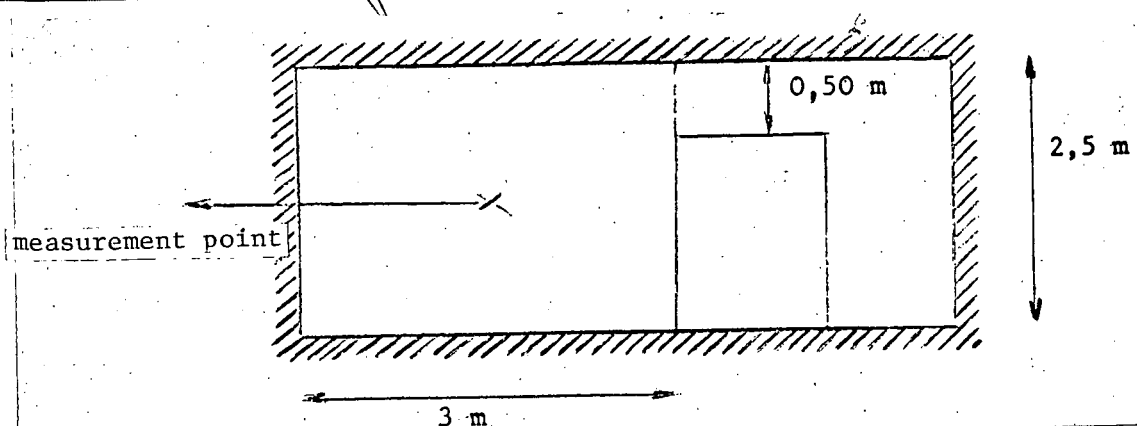
- length 3 meters
- height 2.5 meters

We will call these partitions type 2.

5
4
3
2
1

5
4
3
2
1

1
2
3
4
5



Because of work in progress and the required length of the cables, it was not possible to measure the frequencies of all the partitions.

In addition, certain recordings have a background 500 Hz noise which is too high and were therefore eliminated. The measurements on which results are presented in this report were carried out from the second to the fifth floor, inclusive.

Finally, 18 partitions of type 1 and 6 partitions of type 2 were tested under good conditions.

Figure 3 shows the dispersion of the eigenfrequencies of the two types of partitions with respect to their arithmetic mean. It can be seen that the arithmetic mean of the first eigenfrequency is:

- 47.6 Hz for the type 1 partition
- 41.2 Hz for the type 2 partition.

In addition, the mean square deviation σ is 2.12 Hz for the first type of partition.

5
4
3
2
1

Figure 1. Dispersion of the First eigenfrequency.
 "Pleinclam" plaster tile partition.

Ratio of the eigenfrequencies with respect to their averages
 in Hz.

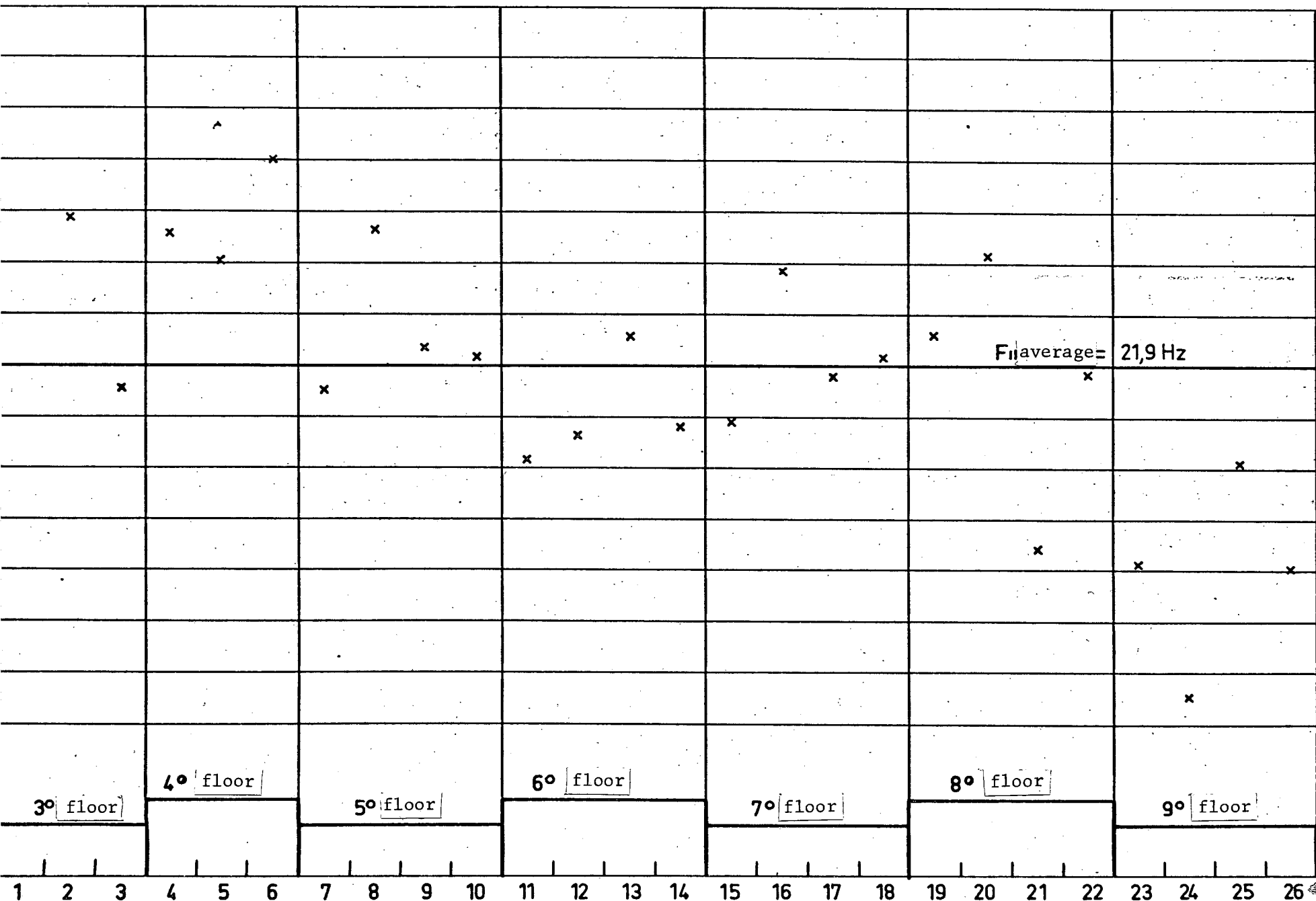


Figure 2. Statistical distribution of eigenfrequencies F 11.
Cumulative Distribution

"Pleinclam" plaster tile partition

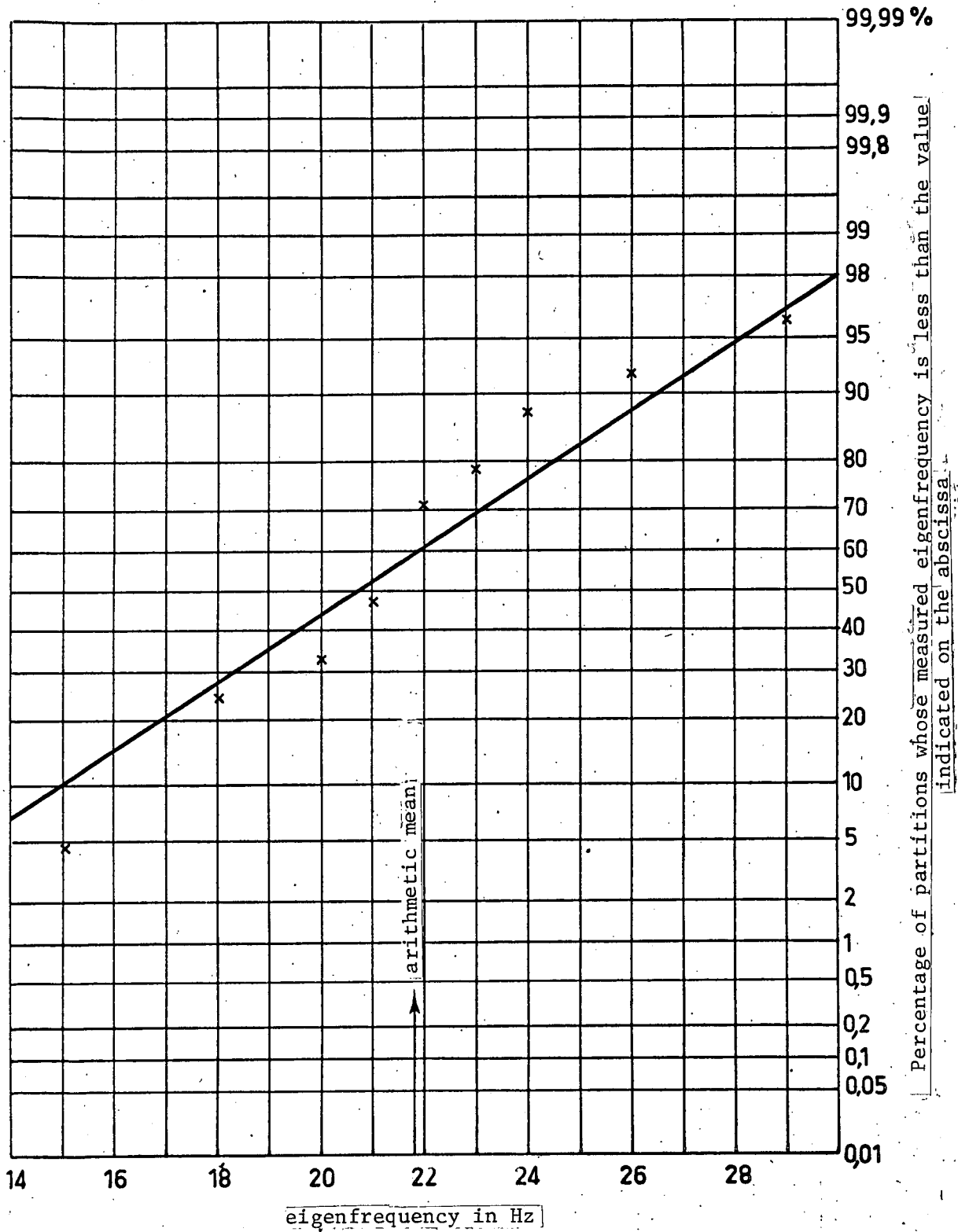


Figure 3. Distribution of the first eigenfrequency

12

Eigenfrequency of

"Placopan" partition

+ measurement partition type 1

Placopan partition

o " " " 2

()

Hz

50

)

45

40

2^o floor

3^o floor

4^o floor

5^o floor

average partition type 1

average partition type 2

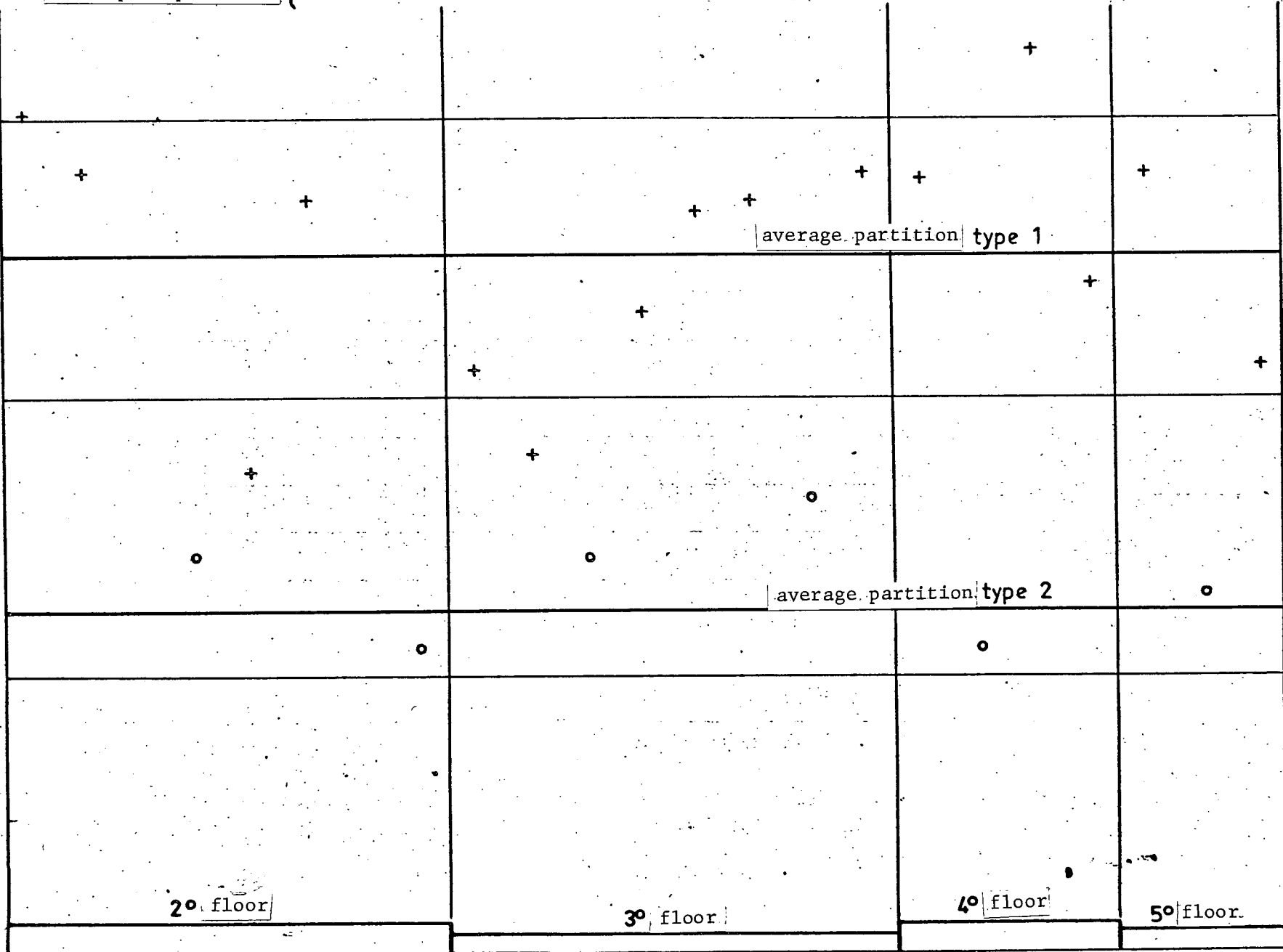


Figure 4 shows the statistical distribution of the eigenfrequencies of the first type of partition. It can be seen that this distribution is essentially Gaussian.

c - "Plaster Brick" Partition

This partition consists of hollow plaster bricks assembled with plaster. They carry a plaster layer on both sides.

The dimensions of a plaster brick are the following:

- total thickness 3.5 cm
- length 40 cm
- width 20 cm

The cross section of such a brick and its internal dimensions are given for the corresponding frequency calculation.

This partition is mounted on the floor by means of a resilient sole, shown in the following diagram:

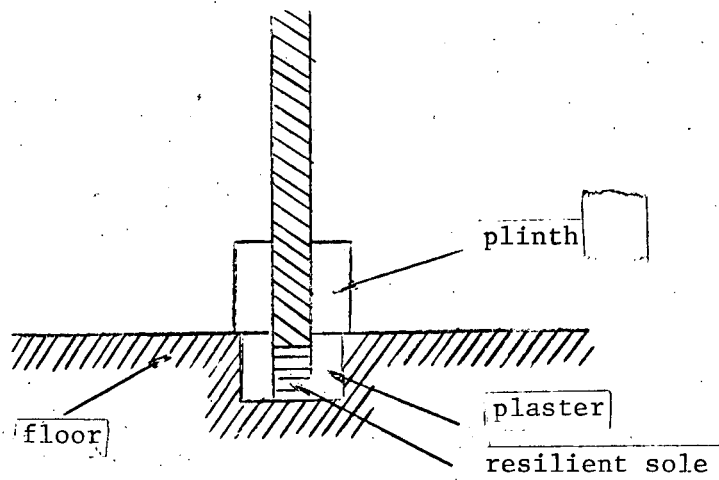


Figure 4. Statistical distribution of the Fall eigenfrequencies

Cumulative distribution

Placopan partition

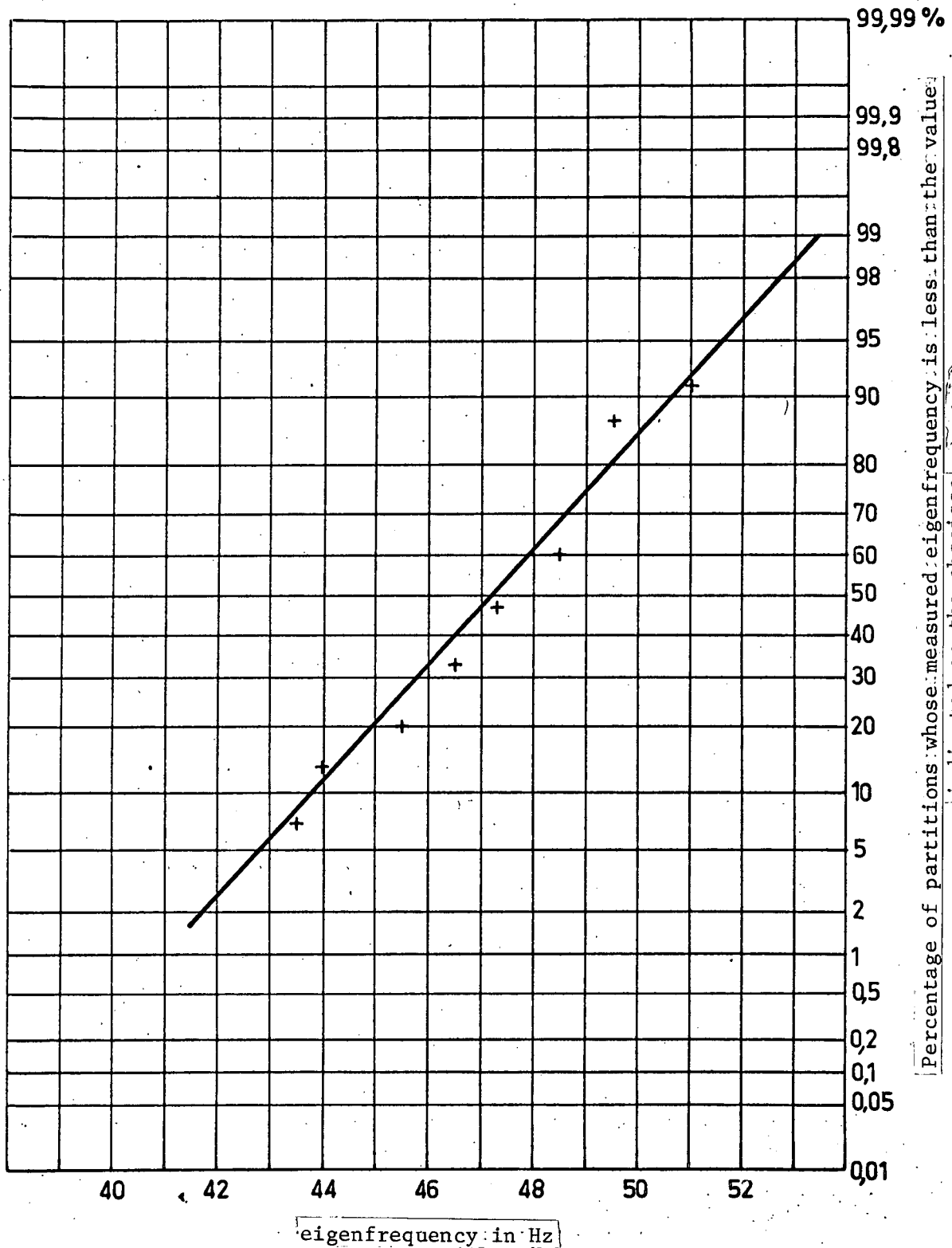
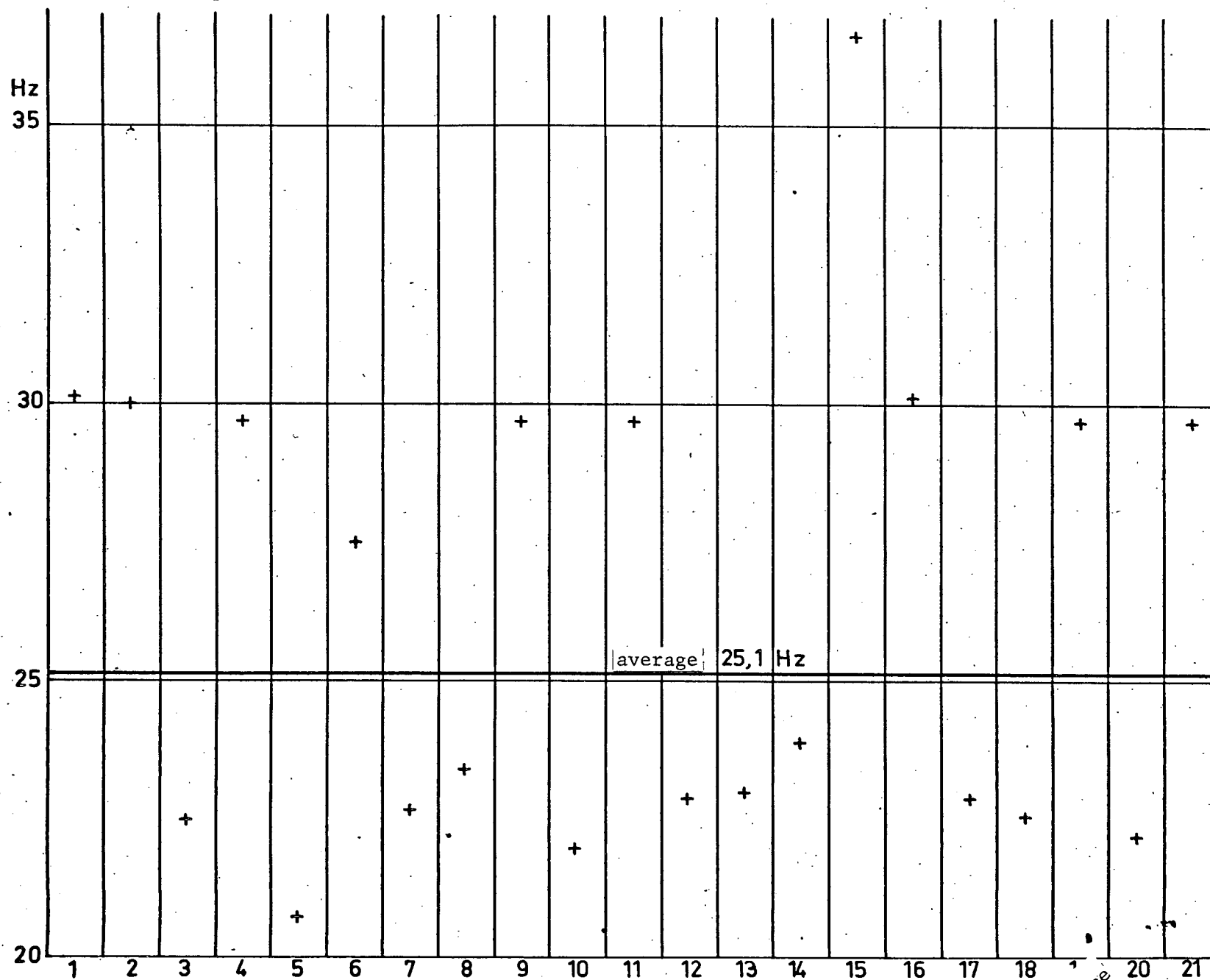


Figure 5. Distribution of the first eigenfrequency
plaster brick partition.

frequency of the partition
plaster brick



1
2
3 The other sides are connected to the structure by plaster sealing.
4

/9

5 The measurements were carried out in a high rise complex having 12 floors with four apartments per floor. A single partition which separates the living room from the kitchen was tested for each apartment. The measurements were carried out on 21 partitions having the following dimensions:

- height 2.5 m

- length 4 m

This panel is held very well along three edges. The fourth edge can be considered as intermediate between clamping and simple support.

The results obtained are given in Figure 5 which show the dispersion of the first eigenfrequency with respect to the arithmetic mean. The calculation shows that:

- the arithmetic mean is 25.1 Hz

- the mean square deviation σ is 4.3 Hz

- the dimensionless damping α , measured on certain recordings, is in the vicinity of 2.5%.

The statistical distribution of the eigenfrequencies is not Gaussian, and was not plotted on the graphs. Examination of the distribution of the eigenfrequencies seems to indicate that it is distributed into two groups; one is made up of frequencies in the vicinity of 30 Hz and the other by those in the vicinity of 23 Hz. An inquiry showed that all the partitions were mounted in the same way, but that the bricks used, which incidentally all have the same dimensions, came from two different populations. These two types of bricks came from two different manufacturing runs and therefore had moduli of elasticity which were considerably different.

d - Window panes

The measurements were carried out in a series of offices on 18 window panes, which all had the identical dimensions, 1.35 x 1.09 m. These window panes having a thickness of 5 mm are installed in aluminum window frames and are held along the edge by rubber.

The results obtained are given:

/10

- in Figure 6, which shows the dispersion of the first eigenfrequency with respect to the arithmetic mean
- in Figure 7 which shows the statistical distribution of the eigenfrequencies. This distribution is essentially Gaussian.

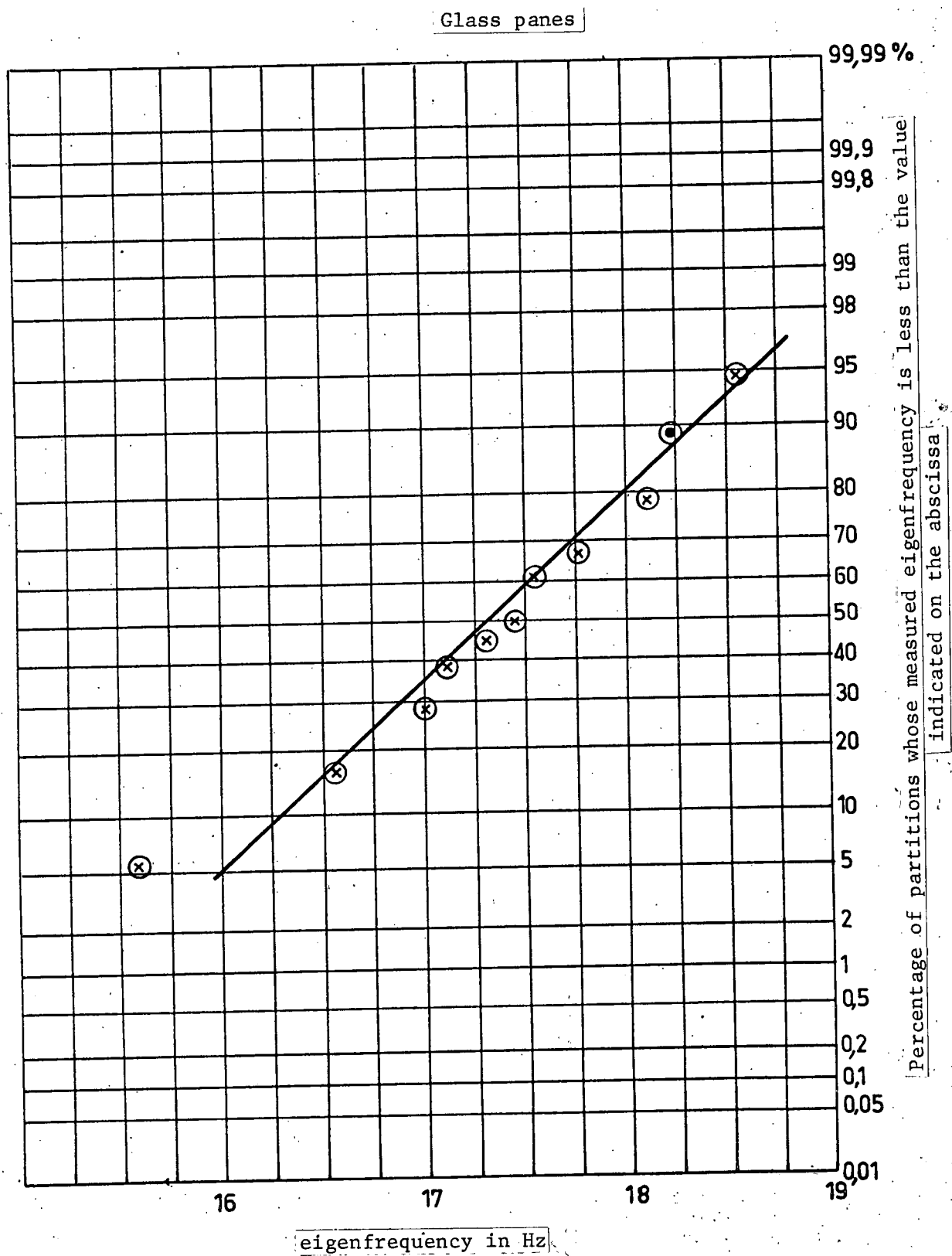
It can be seen that:

- the arithmetic mean of the frequencies is 17.5 Hz
- the mean square deviation σ is 0.80 Hz
- the arithmetic mean of the dimensionless damping α of the first mode is 4.25%. The dampings α measured with 12 window panes are shown in Table I.

Figure 6.

eigenfrequency in Hz							
		+					
	+	+					
						+	+
			+				
		average	17,5 Hz		+	+	
				+			
+	+			+			+
+							
	1 st	2 nd	3 rd	4 th	5 th	6 th	7 th
Laboratory	office	office	office			office	office

Figure 7. Statistical distribution of F 11 eigenfrequencies
cumulative distribution



Remarks:

- In one recording, the eigenfrequency of the 1.3 mode was measured, and was found to be equal to 68 Hz (in good agreement with calculation which gives 69 Hz). The damping of this mode is about 4%.

- It can be seen that the lowest frequencies correspond to an office having a large size. The other windowpanes are arranged in small offices which are essentially identical. The eigenfrequencies vary between 17.1 and 18.55 Hz.

II-4. Theoretical Prediction of the First Eigenfrequency in Comparison with the Experimental Results

a - General Considerations

The first eigenfrequency of a rectangular plate must be calculated. It is assumed that this plate is homegenous, i.e., that its density and its modulus of elasticity are constant at any point of the surface. The rigidity of the plate is calculated for flexture only, which indicates that we do not take the membrane effect into account and that the eigenfrequency does not depend on the amplitude.

This calculation therefore assumes that the following are known with sufficient accuracy:

/11

- the mass of the plate per unit surface in kg/m^2
- the flexture rigidity of the plate in N/m
- the boundary conditions (clamped edges, supported edges, ...)

The mass of a partition or of a window pane per unit surface is generally well known and does not cause any problems.

The rigidity of a partition depends on:

- moduli of elasticity of the materials making up the partition. In effect when the partition is a composite structure (example: cardboard and plaster in the case of Placopan partitions), the moduli of the various components enter the calculation. It is very difficult to obtain accurate values of the moduli of elasticity. The manufactures of the materials do not know them except on rare occasions. In any case, they specify vague values. The deviations of the moduli within different populations of equivalent products are very large. The dispersion among products of the same manufacturing series is frequently also very large.

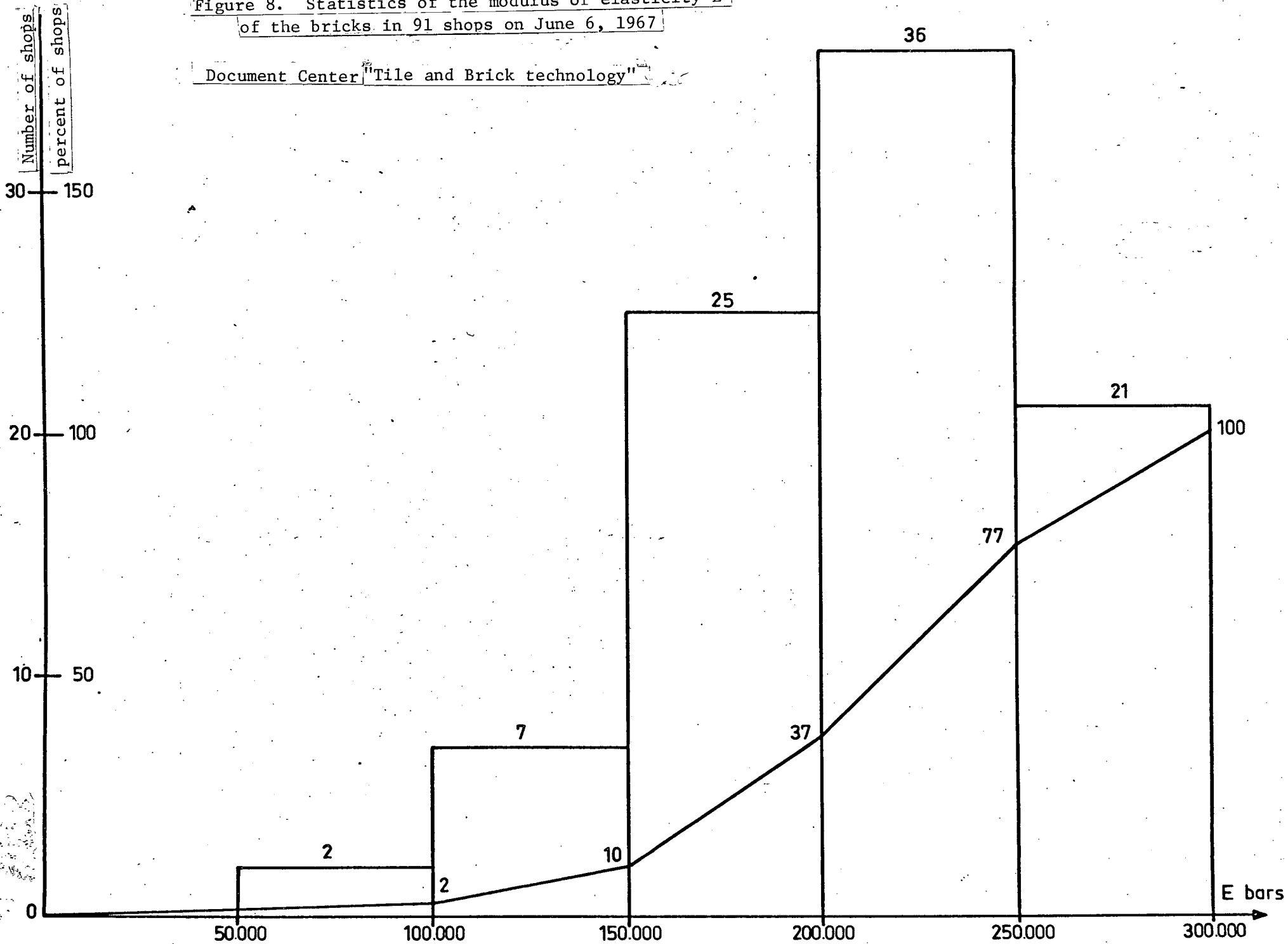
Figure 8 shows the dispersion of the modulus of elasticity of the bricks, (reference "Centre Technique des Tuiles et Briques" [Technical Center for Tiles and Bricks]). Also, whenever possible, we measured the modulus of elasticity directly (example: measurement of the modulus of the elasticity of plaster for one Pleinclam partition -- see Appendix I).

- The transverse dimensions of the walls. If the partition is solid (case of a "Pleinclam" partition), only the thickness enters into the calculation. If the partition is hollow (case of plaster bricks or "Placopan" partition), the thicknesses and the distance to the neutral fiber are important. Given the tolerances on these dimensions (especially for the bricks), inertia variations can result and therefore considerable rigidity variations can result.

The boundary conditions are very poorly defined. Certainly it is this uncertainty which is the greatest error source. In effect, the boundary conditions have a very great influence on the value of the first eigenfrequency. In particular, it will be seen that a partition clamped along the four edges has an eigenfrequency which is practically twice the one for a partition having four simply supported edges. In addition, the phenomenon is complicated by the fact that the four edges of the wall can have varying boundary conditions and that these same boundary conditions can lie in between the ideal theoretical cases. For example, one edge of a partition can be semi-clamped or semi-free. One is limited to ideal boundary conditions in the calculation.

Figure 8. Statistics of the modulus of elasticity E
of the bricks in 91 shops on June 6, 1967

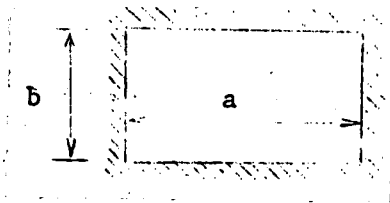
Document Center "Tile and Brick technology"



b - Formulas Used

The types of calculation cases which we studied which resemble the real case the most are the following:

- Rectangular Partitions Clamped Along 4 Edges:



This formula is taken from the document "Multimode response of panels to normal and to traveling sonic booms" by CROCKER (JASA 1967 - V 42 N° 5)

The eigenfrequency of the first mode is given by

$$f_{11} = \frac{1}{2\pi} \left[\frac{\alpha_1^4}{a^4} + \frac{\alpha_1^4}{b^4} + 2 \frac{\psi_1^2}{a^2 b^2} \right]^{1/2} \left[\frac{D}{\rho h} \right]^{1/2} \quad (1)$$

with: a = length of the panel in meters

b = height of the panel in meters

$\alpha_1 = 4.73$

$\psi_1 = 12.3$

ρh = mass per unit area of the partition in kg/m^2

D = coefficient depending on the rigidity of the partition

for a solid partition:

$$D = \frac{E h^3}{12(1-\nu^2)}$$

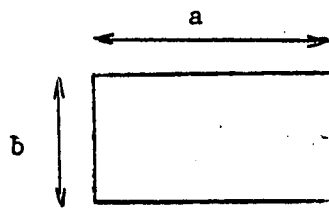
with: E = modulus of elasticity of the material in N/m^2 .

h = thickness of the partition in meters

ν = Poisson coefficient, close to 0.3

- Rectangular Partition Simply Supported Along 4 Edges:

/13

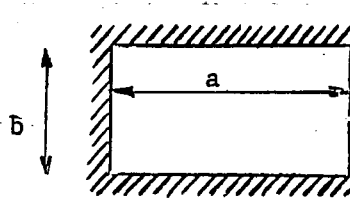


The eigenfrequency of the first mode is:

$$f_{11} = \frac{\pi}{2} \left[\frac{1}{a^2} + \frac{1}{b^2} \right] \left[\frac{D}{\rho h} \right]^{1/2} \quad (2)$$

with the same notation as before.

- Rectangular Partition Clamped Along 3 Edges and Free Along the 4th Edge:



The first eigenfrequency of such a plate approximately corresponds to the eigenfrequency of a plate clamped along four edges having twice the length (2 a) and the same width b. We therefore have:

$$f_{11} = \frac{1}{2\pi} \left[\frac{\alpha_1^4}{16 a^4} + \frac{\alpha_1^4}{b^4} + \frac{\psi_1^2}{2 a^2 b^2} \right]^{1/2} \left[\frac{D}{\rho h} \right]^{1/2} \quad (3)$$

All the coefficients were already defined above. This formula, which corresponds to an inaccurate deformation, leads to an eigenfrequency which is somewhat higher than reality.

- Rectangular Plate Clamped Along 3 Edges and with Simple Support for the 4th Edge:

The first eigenfrequency of such a plate is equal to the eigenfrequency of the second vibration mode of a plate having twice the length and clamped along the four edges. The line of nodes of this mode coincides with the simple support edge of the plate a, b.

We therefore have:

$$f_{11} = \frac{1}{2\pi} \left[\frac{\alpha_2^4}{16 a^4} + \frac{\alpha_1^4}{b^4} + \frac{\psi_1 \psi_2}{2 a^2 b^2} \right]^{1/2} \left[\frac{D}{\rho h} \right]^{1/2} \quad (4)$$

with the new constants:

$$\alpha_2 = 7.85$$

$$\psi_2 = 46.05$$

c - Results of Theoretical Predictions and Comparison with the Experimental Results.

1 - "Pleinclam" Partition

The quantities given for the calculation of the following:

$$a = 4.40 \text{ m}$$

$$b = 2.5 \text{ m}$$

$$(\rho h) = 64.5 \text{ kg/m}^2$$

$$E = 2.4 \cdot 10^9 \text{ pascals (or N/m}^2\text{)}.$$

Since we do not have any accurate data regarding the modulus of elasticity of a "Pleinclam" plaster tile, we measured it in the laboratory (see Appendix I).

Since we are dealing with a solid partition, we have:

$$D = \frac{E h^3}{12(1-\nu^2)}$$

Finally we find:

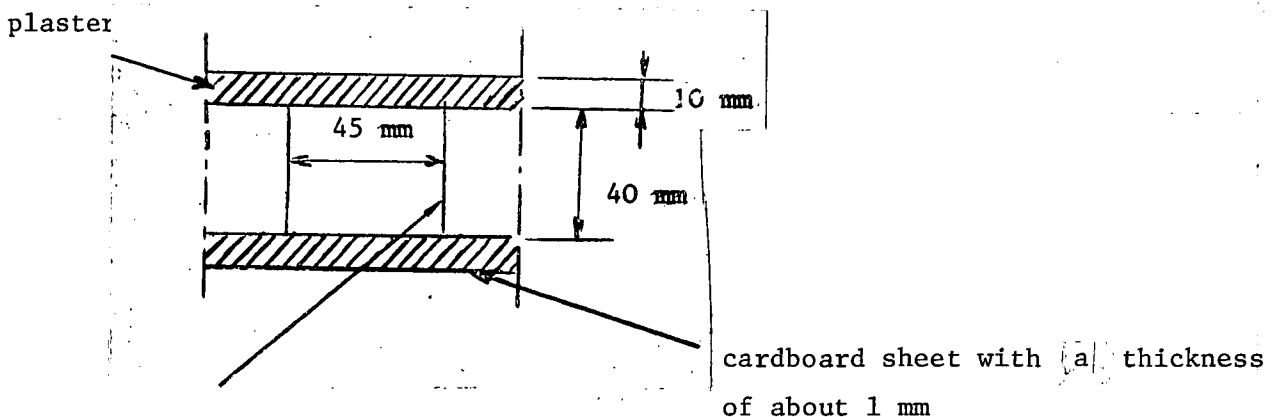
$f_{11} = 11$ Hz for the calculated simply supported partition (Formula 2)

$f_{11} = 22$ Hz for the calculated partition with four clamped edges (Formula 1).

The arithmetic mean of the measured eigenfrequencies is 21.9 Hz. It seems therefore that the four edges are clamped in practice.

2 - "Placopan" Partition

The internal dimensions of the "Placopan" which make up the partitions for which measurements were carried out are the following (see cross-section below):



crossed cardboard sheets which form
a cube (40 mm side length.)

The term D , which appeared in our previous formulas, corresponds to the flexure rigidity of a beam having a length of 1 meter and the cross-section shown above. The flexure rigidity is therefore:

$$D = I.E.$$

I moment of inertia with respect to the neutral fiber
E modulus of elasticity

The total moment of inertia is:

$$\left| \begin{aligned} I &= \frac{2}{3} l \left[(30)^3 - (20)^3 \right] 10^{-9} \text{ with } l = 1 \text{ m} \\ I &= 12,65 \cdot 10^{-6} \text{ m}^4 \end{aligned} \right|$$

The modulus of elasticity E was taken to be equal to $7.5 \cdot 10^9$ pascals, /16
according to information given to us. However, the true modulus of
the elasticity of the tested partitions could be considerably different,
given the fact that no direct measurement could be carried out on the material
which makes up the partitions.

Therefore:

$$D = I E = 12,65 \cdot 10^{-6} \cdot 7,5 \cdot 10^9 = 9,5 \cdot 10^4$$

The given quantities of the calculation are the following:

- a = 3.15 m
- b = 2.50 m
- (ρh) = 18.6 kg/m²
- D = 9.5 · 10⁴

The results are as follows:

- f_{11} = 30.4 Hz for the partition with four simply supported edges
- f_{11} = 52.6 Hz for the partition with three clamped edges, one supported edge
- f_{11} = 56.2 Hz for the partition with four clamped edges.

The arithmetic mean of the measured eigenfrequencies is equal to 47.6 Hz
for the first type of partition.

It is difficult to determine whether the deviations with the calculation
are due to non-ideal boundary conditions (partial clamping, for example) or

because the selected value of the modulus of the elasticity is slightly wrong.

The second type of partition has an almost free edge and slightly smaller dimensions

$$(a = 3 \text{ m}, b = 2.5 \text{ m})$$

The calculation results in:

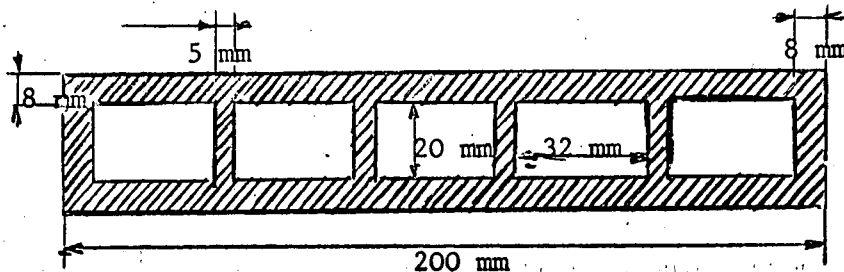
/17

$$f_{11} = 46.3 \text{ Hz with three clamped edges and one free edge.}$$

The arithmetic mean of the measured eigenfrequencies of the second type of partition is equal to 41.2 Hz.

3 - Plaster Brick Partition

The cross-section of a plaster brick shows the edges which must be taken into account in order to calculate the moment of inertia:



In the calculation of the inertia, the inertia due to the struts is not taken into account, because it is very small with respect to those of the flanges.

For a section of length l , the inertia is equal to:

$$I = \frac{2}{3} l (18^3 - 10^3) 10^{-9} = 3,21 l 10^{-6} \text{ m}^4$$

Therefore, if $l = 1 \text{ m}$:

$$I = 3.21 \cdot 10^{-6} \text{ m}^4$$

We have assumed an average modulus of elasticity:

$$E = 2.3 \cdot 10^{10} \text{ N/m}^2$$

Under these conditions, the term D, corresponding to the flexure rigidity, is equal to:

$$D = I E = 3.21 \cdot 10^{-6} \cdot 2.3 \cdot 10^{10} = 7.35 \cdot 10^4$$

The given quantities in the calculation are the following:

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$$a = 4 \text{ m}$$

$$b = 2.5 \text{ m}$$

$$(\rho h) = 32.8 \text{ kg/m}^2, \text{ obtained directly by weighing}$$

$$D = 7.35 \cdot 10^4$$

The results of the calculation are the following:

$$f_{11} = 31.8 \text{ Hz for four clamped edges}$$

$$f_{11} = 30.6 \text{ Hz with three clamped edges and the fourth simply supported}$$

$$f_{11} = 16.55 \text{ Hz with the four edges simply supported.}$$

The arithmetic mean of the measured eigenfrequencies corresponding to this is 25.1 Hz. It is difficult to determine whether the deviations with the calculation are due to hypotheses regarding the boundary conditions or because of an erroneous value of the modulus of elasticity.

4 - Window Panes

The computational data are the following:

$$a = 1.35 \text{ m}$$

$$b = 1.09 \text{ m}$$

$$E = 6.3 \cdot 10^{10} \text{ N/m}^2$$

$$\rho = 2500 \text{ kg/m}^3$$

$$h = 5 \cdot 10^{-3} \text{ m}$$

We have selected average values for E and ρ , which are commonly assumed for glass.

Finally, we find:

$L_{11} = 16.5 \text{ Hz}$ for the windowpane calculated for simple support.

This result is in quite good agreement with the results of the measurement; /19 the arithmetic mean of the eigenfrequencies found by experiment are 17.5 Hz.

The calculation shows that the additional rigidity introduced by the air enclosed in the room is negligible, given the volume of each room.

III. Conclusions

This study has given us information on the dynamic character-istics of several types of light construction elements which are sensitive to sonic boom.

The comparative results are as follows:

- first eigenfrequency: (measured arithmetic mean)

$f_{11} = 17.5 \text{ Hz}$ for the window panes

$f_{11} = 21.9 \text{ Hz}$ for the partitions made of solid plaster tiles

$f_{11} = 25.1 \text{ Hz}$ for partitions made of plaster brick

$f_{11} = 47.6 \text{ Hz}$ for "Placopan" partitions.

- Damping α of the first mode:

$\alpha = 2.5\%$ for plaster brick partitions

$\alpha = 3\%$ for "Placopan" partitions

$\alpha = 4.25\%$ for window panes.

- Dispersion:

In order to compare the various measured elements, given the different eigenfrequencies, we can define a dispersion index d defined by the ratio:

$$d = \frac{\sigma}{f_{11}} \quad \left| \begin{array}{l} \sigma \text{ standard deviation of the eigenfrequency} \\ f_{11} \text{ arithmetic mean} \end{array} \right.$$

We then obtain the following classifications:

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$$d = \frac{2.1}{47.6} = 4.4\% \text{ for the first type of "Placopan" partition}$$

$$d = \frac{0.8}{17.5} = 4.57\% \text{ for the windowpanes}$$

$$d = \frac{4.3}{25.1} = 17.1\% \text{ for the plaster brick partitions}$$

$$d = \frac{4.5}{21.9} = 20.5\% \text{ for the "Pleinclam" tile partitions.}$$

The index d increases in proportion to the dispersion of the frequency.

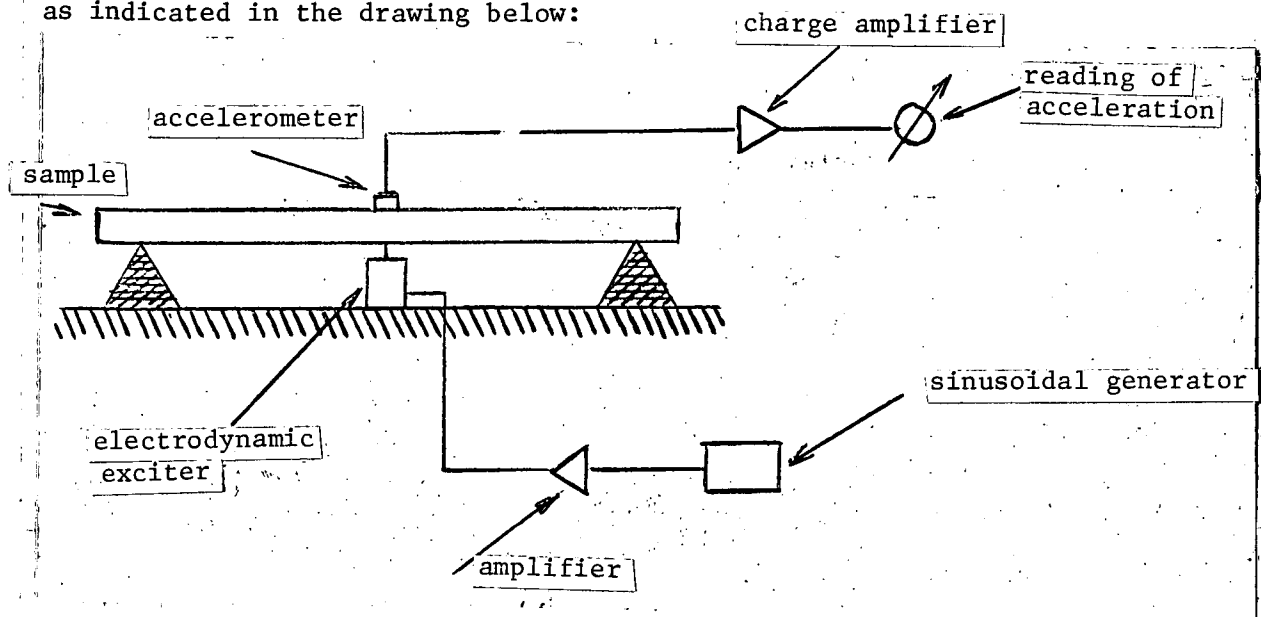
It should also be noted that the elements which are most sensitive to the sonic boom are those for which the first eigenfrequency is sufficiently low, so that they will be sufficiently in agreement with the sonic booms of the "Mirage" or "Concorde" type. These sonic booms can affect the elements under consideration either directly or indirectly (penetration through an open window).

In this regard, if it seems difficult to find partitions having eigen-frequencies less than 15 Hz, it is possible to find large glassed-in bays which have resonance frequencies less than 10 Hz.

On the other hand, this method has made it possible to verify our theoretical predictions, which have proven to be reliable. This is especially true if we know the boundary conditions and modulus of elasticity of the elements under consideration with sufficient accuracy.

DETERMINATION OF THE YOUNG MODULUS FROM A SAMPLE OF "PLEINCLAM" PLASTER TILE

The measurement of the eigenfrequency f_{11} of the sample is carried out as indicated in the drawing below:



The first eigenfrequency is given by:

$$f_{11} = \frac{\pi}{2l^2} \cdot \frac{E I}{S \rho}$$

with E = Young modulus

I = moment of inertia of the section

S = section of the bar

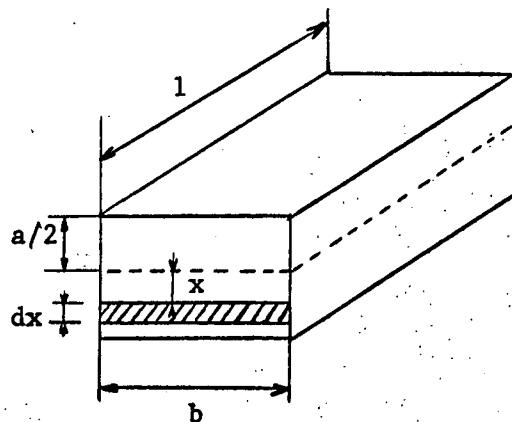
ρ = volume mass

l = length of the bar between supports.

- Calculation of the moment of inertia:

$$I = \int x^2 dS$$

$$I = 2b \int_0^{a/2} x^2 dx = \frac{b a^3}{12}$$



The inertia is calculated from the values of a and b

$$a = 0.07 \text{ m}$$

$$b = 0.10 \text{ m}$$

- the quantity $S\rho$ is known:

$$s = a \times b$$

$$\rho = 920 \text{ kg/m}^3$$

- the length l is equal to:

$$l = 0.66 \text{ m}$$

- the frequency f_{11} is measured during a frequency sweep with constant force. The corresponding frequency at the maximum acceleration is the desired frequency f_{11} .

We then derive the modulus of elasticity E :

$$E = \frac{4 \rho S l^4 f_{11}^2}{\pi^2 I}$$

We find: $E = 2.4 \cdot 10^9$ Pascals.

TABLE I - DAMPING α OF THE FIRST MODE FOR 12 WINDOW PANES

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α	α
3 %	4.8%
3 %	5.0%
3.3%	5.0%
4.0%	5.2%
4.0%	5.3%
4.1%	5.4%

The damping α therefore varies between 3 and 5.4%. The dispersion of the measurements is therefore considerable, which is very often the case in measurements of damping.

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